A Fast Computation of Wound Rotor Induction Machines Based on Coupled Finite Elements and Circuit Equations under a First Space Harmonic Approximation

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The paper presents a fast method to compute wound rotor induction machines in steady state. Coupled time-harmonic FE-circuit equations are used under a first space harmonic approximation for the air-gap magnetic field. It is shown that only 4 magnetostatic FE computations are necessary to compute the machine performances for a wide range of operating speeds. The performances comparison to a conventional complex magnetodynamic FE analysis shows the effectiveness of the proposed approach.

***Index Terms*—Circuit equations, Finite element analysis, Fourier series, Induction machine, Wound rotor**

I. Introduction

Wound rotor induction machines (WRIM) are nowadays widely used in wind turbines as well as in flywheels, pumps and fans systems. Indeed, the doubly fed configuration (DFIM or DFIG) of WRIM is well known for its outstanding variable speed capability, adjustable power factor and reduced converter rating [1]. The design of WRIM can be done using a variety of methods. The concepts of electric and magnetic loadings together with manufacturers past experience allows an initial sizing of the machine [2]. Then, a precise finite element (FE) analysis is carried out in a final design stage [1]-[2]. Unfortunately, the exclusive use of finite elements in the design process of induction machines leads to a very long computation time.

We propose in this paper an approach based on FE-circuit analysis that allows a fast and precise computation of WRIM performances in steady state. The magnetic field is truncated so only the principal air-gap space harmonics are considered. A similar approach has been successfully used for the computation of squirrel-cage induction motors [3]-[5]. A FE computation is needed for each value of the slip frequency in the rotor bars. In this paper, it will be shown that only four FE magnetostatic computations are necessary to determine the WRIM performances for a wide range of speed operation.

II. The Electromagnetic Model

A magnetic vector potential formulation is used under the usual plane 2D approximation. The background of the electromagnetic model is the same as the one described in [5]-[6] for squirrel cage induction motors. The machine is split into two domains of resolution noted D_s and D_r, Fig.1. Both domains include the air-gap domain D_g. The ferromagnetic materials are considered linear. However, the method can also consider the magnetic saturation in an averaging sense [6].

We assume that the machine is supplied from a balanced three-phase sinusoidal system of currents and only one time pulsation is present in the source currents. By considering that only the first space harmonic of p pole-pairs exists in the air gap, the vector potential is expressed as follows

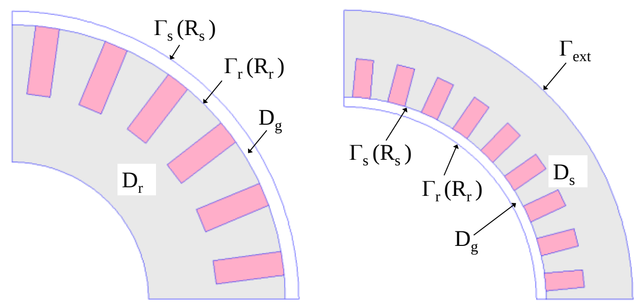

Fig. 1. Stator (D_s) and rotor (D_r) domains

$$a_s(P,t) = \sqrt{2}Re\left[\left(\overline{X_s}(P) + \overline{C_s}.\overline{A_s}(P)\right)\exp(j\omega_s t)\right] \quad in\ D_s \qquad (1)$$

$$a_r(P',t) = \sqrt{2}Re\left[\left(\overline{X_r}(P') + \overline{C_r}.\overline{A_r}(P')\right)\exp(j\omega_r t)\right] \ in\ D_r \qquad (2)$$

Where ω_s and ω_r are the electrical pulsations in the stator and the rotor domains, respectively. $\overline{X}$ and $\overline{A}$ are complex elementary vector potentials (the indices r and s stand respectively for rotor and stator). P and P' are points defined in polar coordinate systems attached to the stator and the rotor domains such as $\theta = \theta' + \Omega t$ (Ω is the rotor velocity).

In addition to (1) and (2), a_s and a_r must coincide everywhere in the air-gap. To do so, it is sufficient to ensure the following continuity relations in D_g

$$\begin{cases} a_s(R_s,\theta,t) = a_r(R_s,\theta',t) & on\ \Gamma_s \\ a_s(R_r,\theta,t) = a_r(R_r,\theta',t) & on\ \Gamma_r \end{cases} \qquad (3)$$

Indeed, a_s and a_r are harmonic functions in the air-gap (they are solution of the Laplace equation) so their equality on the air-gap boundaries Γ_s and Γ_r allows their coincidence everywhere in the air-gap. Furthermore, this paper will consider only the principal p pole-pairs space harmonic in the air-gap. In steady state operation, $\omega_s = p\Omega + \omega_r$ and the slip is $s = (\omega_s - p\Omega)/\omega_s = \omega_r/\omega_s$. In classical WRIM, ω_r is due to the induced currents in the short-circuited rotor windings. In DFIM or DFIG, ω_r is imposed by an external rotor ac supply.

The complex constants $\overline{C_s}$ and $\overline{C_r}$ correspond to the Fourier series coefficients of the first space harmonic (p pole-pairs) of the vector potentials in the air-gap. The aim here is to determine these coefficients together with the elementary vector potentials $\overline{X}$ and $\overline{A}$ to get the solution using (1) and (2).

A. *Computation of* $\overline{X_s}$ *and* $\overline{A_s}$

$\overline{X_s}$ corresponds to the source problem. The stator windings are supplied by a unity 3-phase current. We set $\overline{X_s} = 0$ on Γ_r and Γ_{ext}. We solve by FE the Laplace (in the iron parts and the air-gap) and Poisson (in the slots) partial differential equations (PDEs). Then we compute the pth harmonic Fourier coefficient noted $\overline{\mu_s}$ on Γ_s. We also compute the magnetic flux noted $\overline{\varphi_{sX}}$ in phase 1 for example (the choice of the phase is arbitrary).

$\overline{A_s}$ corresponds to the rotor armature reaction. The 3-phase stator windings are not supplied. $\overline{A_s} = 0$ on Γ_{ext} and $\overline{A_s} = \exp(jp\theta)$ on Γ_r We solve by FE the Laplace PDEs (iron parts, slots and air-gap). Then we compute the pth harmonic Fourier coefficient noted $\overline{\lambda_s}$ on Γ_s. We also compute the magnetic flux noted $\overline{\varphi_{sA}}$ in phase 1.

B. *Computation of* $\overline{X_r}$ *and* $\overline{A_r}$

The rotor windings are supplied by a unity 3-phase current. We set $\overline{X_r} = 0$ on Γ_s. We solve by FE the Laplace (iron parts and air-gap) and Poisson (slots) PDEs. Then we compute the pth harmonic Fourier coefficient noted $\overline{\mu_r}$ on Γ_r. We also compute the magnetic flux noted $\overline{\varphi_{rX}}$ of phase 1.

The 3-phase rotor windings are not supplied. We set $\overline{A_r} = \exp(jp\theta')$ on Γ_s We solve by FE the Laplace PDEs (iron parts, slots and air-gap). Then we compute the pth harmonic Fourier coefficient noted $\overline{\lambda_{r0}}$ on Γ_r. We also compute the magnetic flux noted $\overline{\varphi_{rA}}$ of phase 1.

C. *Computation of* $\overline{C_s}$ *and* $\overline{C_r}$

The first step is to calculate the effective rotor current since its value was assumed unity when computing $\overline{X_r}$. To do so, we use the rotor phase circuit equation

$$\overline{V_r} = (r_r + j\omega_r l_{rew})\overline{I_r} + j\omega_r(\overline{\varphi_{rA}} + \overline{I_r}.\overline{\varphi_{rX}}) \quad (4)$$

$\overline{V_r}$ is the phase rotor voltage (equals to 0 in usual short-circuited rotor windings), r_r and l_{rew} are the rotor phase resistance and end-winding phase inductance, respectively. The actual vector potential $\overline{X_r}$ (and also all the quantities related to it) are obtained by multiplying the one computed for unity rotor current by $\overline{I_r}$ computed using (4).
Hence, the actual Fourier coefficient for the whole rotor problem (superposition of $\overline{X_r}$ and $\overline{A_r}$) is then written as $\overline{\lambda_r} = \overline{\lambda_{r0}} + \overline{I_r}.\overline{\mu_r}$.
Now, we are able to compute $\overline{C_s}$ and $\overline{C_r}$ using (3). This leads to solve the following two algebraic complex equations

$$\begin{cases} \overline{C_r} - \overline{\lambda_s}.\overline{C_s} = \overline{\mu_s} \\ \overline{\lambda_r}.\overline{C_r} - \overline{C_s} = 0 \end{cases} \quad (5)$$

The stator being usually supplied by a voltage source rather than by a current source, the stator current of the machine is then obtained via the stator phase circuit equation as follows

$$\overline{I_s} = V_s/(r_s + j\omega_s l_{sew} + \overline{Z}) \quad (6)$$

Where $\overline{Z} = j\omega_s(\overline{\varphi_{sX}} + \overline{C_s}.\overline{\varphi_{sA}})$ is the operational impedance of the machine (obtained with the unitary stator current), r_s and l_{sew} are the stator phase resistance and end-winding phase inductance, respectively.
It is clear that only 4 FE complex-magnetostatic computations are required to have the solution for any slip value (the slip only appears in the rotor circuit equation (4)).

III. Application Example

The proposed method has been tested on a short-circuited rotor WRIM rated at 100 kW, 50Hz, 400 V delta and p=3. The nominal speed is 980 rpm (s=2%).
Fig.2. shows the computed electromagnetic torque and per phase rms stator current for s=0:0.1. It can be seen that the obtained results are in good agreement with those obtained using a full time-harmonic FE model of the whole machine.
For higher slip values, this concordance is not so good because of the influence of higher space harmonics. These issues as well as the saturation effect will be discussed in the full version of the paper.
The overall computation time is about 6s using the proposed method (4 FE computations). For the full time-harmonic model, the computation time for 11 slip values is about 25s.

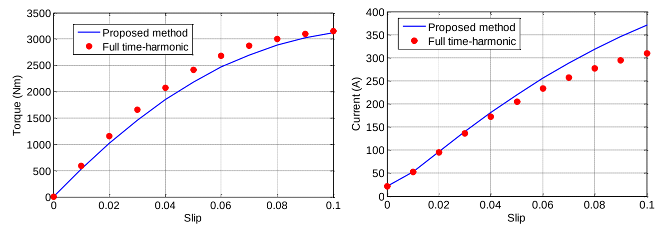

Fig. 2. Torque and stator current vs. slip curves

IV. Conclusion

The presented method, based on the first space harmonic approximation and coupled FE-circuit equations of WRIM, is very fast and accurate at nominal operation. The consideration of higher space harmonics and the magnetic saturation will improve the method so as to constitute a robust an accurate tool for WRIM modeling and optimization.

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